647510 **BSCabsCC6020** **Seat No :________**

B.Sc. Semester - 6 (*CBCS*) Examination

March/April-2024 (OLD COURSE)

MATHEMATICAL ANALYSIS-2 AND ABSTRACT ALGEBRA -2(CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que-1(A) Answer any one out of two. **(07)**

(1) X is metric space and K is compact subset of X and E is infinite subset of K then show that atleast one limit point of E exists in K.

(2) Show that continuous image of compact set is compact.

Que-1(B) Answer any one out of two. **(04)**

(1) X is metric space and E_1 and E_2 be two connected subsets of X and $E_1 \cap E_2 \neq \phi$ then show that $E_1 \cup E_2$ is connected.

(2) Show that the set [0,1] is uncountable.

Que-1(C) Answer any one out of two. **(03)**

(1) Check whether $\mathbb{R}$ -{1} is connected or not where $\mathbb{R}$ is set of all real numbers.

(2) Check whether the following subsets of $\mathbb{R}$ are compact or not.

(i) {1,3,5,7,…………} (ii) $(0,4) \cap (4,9)$ (iii) $\left\{\frac{1}{n} / n \in \mathbb{N}\right\} \cup \{0\}$

Que-2(A) Answer any one out of two. **(07)**

(1) State and prove first shifting theorem and find L[cosh(at)].

(2) Prove: If $L\{f(t)\} = \bar{f}(s)$ then $L\{e^{at}f(bt)\} = \frac{1}{b}\,\bar{f}\left(\frac{s-a}{b}\right), b > 0$. Using this property find L[sinh(at) sin(at)].

Que-2(B) Answer any one out of two. **(04)**

(1) Find : $L^{-1}\left[\frac{3s+5}{(s+1)}\right]$

(2) Find : L[tsin(at)] using derivative of Laplace transform.

Que-2(C) Answer any one out of two. **(03)**

(1) Find : L[x^2sin(hx)].

(2) Find : $L^{-1}\left[\frac{3s+5}{(s+1)^4}\right]$

Que-3(A) Answer any one out of two. **(07)**

(1) Prove that if $L\{f(t)\} = \bar{f}(s)$ then $L\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\,\bar{f}(s)$.

Using this result find $L^{-1}\left[\frac{1}{s(s+a)^3}\right]$.

(2) If $L\{f(t)\} = \bar{f}(s)$ and $\frac{f(t)}{t}$ has Laplace transform then

prove that $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \bar{f}(s)ds$. Using this result Find $L\left[\frac{\sin(t)}{t}\right]$.

Que-3(B) Answer any one out of two. **(04)**

(1) Using convolution theorem find $L^{-1}\left[\frac{s}{(s^2+4)^2}\right]$.

(2) Find : $L\left[\frac{1-e^t}{t}\right]$.

Que-3(C) Answer any one out of two. **(03)**

(1) Find $L[t^n]$ Where n is non-negative integer.

(2) Find : $L^{-1}\left[\frac{1}{s(s-1)}\right]$.

Que-4(A) Answer any one out of two. **(07)**

(1) Define Homomorphism and prove that if $H \leq G$ then $\phi(H) \leq G'$ where (G,*) and (G',*') be two groups and $\phi : G \to G'$ is homomorphism.

(2) Prove that cancellation law exists in a ring R iff R has no zero divisors.

Que-4(B) Answer any one out of two. **(04)**

(1) Show that every field is an integral domain. Is the converse true ? Explain.

(2) For non-zero polynomial $f(x),g(x) \in D[x]$ show that [f(x)g(x)]=[f(x)]+[g(x)].

Que-4(C) Answer any one out of two. **(03)**

(1) Illustrate with an example of subring of ring which is left ideal but not right ideal.

(2) Find zeros of $f(x)=4x^2+3x+2 \in \mathbb{Z}_5[x]$.

Que-5(A) Answer any one out of two. **(07)**

(1) State and prove division algorithm theorem of polynomial.

(2) Define irreducible polynomial.Show that $f(x) = x^4-2x^2+8x+1$is irreducible over Q.

Que-5(B) Answer any one out of two. **(04)**

(1) Prove that factors of x^4+3x^3+2x+4 in $Z_5[x]$ is $(x-1)^3(x+1)$.

(2) Define : Monic polynomial, gcd of polynomial.

Que-5(C) Answer any one out of two. **(03)**

(1) Find G.C.D. of $f(x),g(x) \in \mathbb{Z}_5[x]$. Where $f(x)=x^3+3x^2+3x+3$ and $g(x)=4x^3+2x^2+2x+2$.

(2) If a is a non-zero Quaternion number then find a^{-1}. Also find $[(1+3i)(4j+3k)]^{-1}$

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